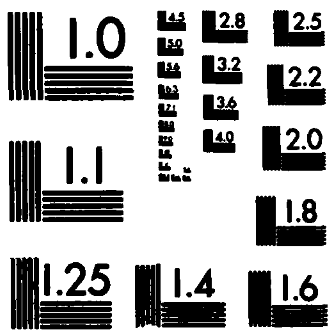
AD-A133 591 DISPLAYING COLORS OF SPECIFIED CHROMINANCE ON A COLOR GRAPHICS DISPLAY..(U) ANACAPA SCIENCES INC SANTA BARBARA CA J C GUTMANN ET AL. DEC 82 AS-TR-459-4 1/1

UNCLASSIFIED DAVAA-TR-81-0089-4 DAAK80-81-C-0089 F/G 9/2 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A133591

DISPLAYING COLORS OF SPECIFIED CHROMINANCE ON A COLOR GRAPHICS DISPLAY

Technical Report TR 459-4

James C. Gutmann

Steven P. Rogers

Submitted to:

ADVANCED SYSTEMS DIVISION (DAVAA-F)
U.S. Army Avionics R&D Activity
Fort Monmouth, New Jersey 07703

December 1982

ANACAPA SCIENCES, INC.
P. O. DRAWER Q,
SANTA BARBARA, CA 93102
TELEPHONE (805) 966-6157

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER DAVAA-TR-81-0089-4	2. GOVT ACCESSION NO. AD-A132591	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Displaying colors of specified chrominance on a color graphics display		5. TYPE OF REPORT & PERIOD COVERED INTERIM TECHNICAL REPORT February --December 1982
		6. PERFORMING ORG. REPORT NUMBER 459-4
7. AUTHOR(s) Gutmann, James C., and Rogers, Steven P.		8. CONTRACT OR GRANT NUMBER(s) DAAK80-81-C-0089
9. PERFORMING ORGANIZATION NAME AND ADDRESS Anacapa Sciences, Inc. 901 Olive Street, P.O. Drawer Q Santa Barbara, CA 93102		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Procurement Br. A, Procurement Director USACECOM, Fort Monmouth, N.J. 07703		12. REPORT DATE December 1982
		13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) USA Avionics R&D Activity Advanced Support Division Fort Monmouth, N.J. 07703		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

color
chrominance characterization
computer graphics display
display systems
human factors engineering
luminance output of a CRT

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A technique for displaying colors of specified chrominance is reviewed. The technique makes use of models of the bits-to-luminance transfer function of the phosphors of a color graphics system CRT. Measurements of the transfer function of a graphics system were made. Second order polynomial models were found to provide a good fit of the bits-to-luminance relationship. A two-part model or spliced fit approach yielded the most accurate predictions of luminance. The algebraic basis of the technique is reviewed as are the limitations.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Cont'd

of the technique. Example solutions for graphics system inputs yielding colors of specified chrominance are provided. The technique provides a method for control of color mixing and for quantitative specification of colors. The incorporation of models of the bits-to-luminance transfer function simplified the procedure used to solve for color graphics system bit values. The technique developed need not be limited to use with color CRTs. Once the input-to-luminance output relationship of any display has been measured and characterized, and once the color coordinates of the system's primaries are known, the technique for determining system inputs can be applied. The only restriction is that the CIE color coordinates of the display system color mixing primaries must not vary as a function of luminance output.

ACKNOWLEDGEMENTS

The development of a technique for displaying colors of specified chrominance required skill and attention to detail on the part of several persons. The authors are indebted to Ms. Nicole Constable, Ms. Leah J. Ralstin, and Mr. James Simon for their assistance in completing this task. The authors wish to gratefully acknowledge the insightful guidance and support of Dr. Norman K. Shupe, Acting Chief of the Research and Technology Division, Avionics Research and Development Activity at Fort Monmouth, New Jersey.

FOREWORD

This report presents the results of research conducted under Contract No. DAAK80-81-C-0089, issued by the U.S. Army Communications-Electronics Command (CECOM) in support of the Avionics R&D Activity (AVRADA) at Fort Monmouth, New Jersey. AVRADA is currently developing a computer-generated topographic display (CGTD) system for use by Army aviators. A CGTD system with a color cathode ray tube (CRT) can be expected to solve existing problems with map handling, scale, and content, and could also provide dramatic improvements in cartographic support, map-oriented computations, and aviator-map interactions.

The CGTD presents the Army aviator with a diverse range of information. As is true with hard copy military topographic maps, the use of color codes is expected to aid visual search. While a great deal is known about color vision and about the effects of color codes on visual search, neither design handbooks nor the psychophysical literature provide specific guidance about the set of colors that should be used for a display as complex as CGTD. To select a set of colors appropriate for CGTD, a series of studies of color coding schemes must be undertaken.

A first step in preparing studies of color coding schemes involves establishing a procedure for displaying colors of known chrominance, as specified by standard color coordinates (such as Commission Internationale de l'Eclairage CIEXYZ coordinates). This report contains the description of a procedure for displaying colors of known chrominance as specified by CIEXYZ coordinates. The procedure makes use of models of a color graphics system intensity (bits) to luminance transfer function.

TABLE OF CONTENTS

LIST OF FIGURES

Figure		Page

LIST OF TABLES

INTRODUCTION

The use of color cathode ray tube (CRT) or television displays in aircraft cockpits is becoming more widespread. The typical justification for using color displays is that color serves as a very effective coding dimension. While recommendations to use color coding to improve visual search performance are often given, research results show that color coding may improve, have no effect on, or interfere with visual search performance (as reviewed by Christ, 1975, 1977). Krebs and Wolf (1977) have argued that color coding provides the greatest benefit for cluttered display search when the target color is known, and when only a few levels of the color code are present. In summarizing the relevant literature on color codes, Krebs and Wolf (1977, p. 12) recommend that no more than five different colors be presented on electronic displays. Additionally, color contrast is an important consideration. Carter (1980, Chapter 4) has presented evidence that the detrimental effects of the number of background items on search time can be overcome by providing sufficient color contrast among the colors of a color code.

Many cockpit CRTs, including computer generated topographic displays (CGTD), present complex and cluttered scenes. CGTDs present many types of information: geographical, cultural, navigational, and tactical. Color coding should benefit the operator, as long as an appropriate choice of colors has been made. Selecting the appropriate colors and levels of color contrast to be used is, however, a difficult task. CGTDs are relatively new devices, hence there is no established or widely used CGTD color coding scheme. As indicated by Snyder (1980) the proliferation of color displays has preceded our ability to model the perception of color contrast. Most of the psychophysical data pertaining to the perception of color contrast are obtained from experiments conducted near the threshold of discrimination. Display designers, however, are most concerned about the perception and scaling of suprathreshold color contrast. A level of color contrast must be selected which virtually ensures that the operator can make the necessary discriminations rapidly and accurately. Design guides such as MIL-STD-1472C either provide very general recommendations, or provide specifications for a limited color code typically found in an aviation environment. These general recommendations and specifications are too limited to help the

CGTD designer determine how much luminance and chrominance contrast should be provided between symbols and a background and between similar terrain features. Similarly, the general recommendations are not suitable for determining the shades of color to be used to code terrain elevation.

In the absence of a specific guideline, the display designer can: 1) survey the psychophysical and the human factors literature and apply relevant results, 2) characterize the colors used on hardcopy maps and use similar colors for CGTDs, 3) use such metrics of color contrast as there are, or 4) make some observations of a prototype CGTD to obtain a usable set of colors. Regardless of the method used to obtain an initial set of colors, at some point the adequacy of a prototype CGTD will be evaluated to determine whether or not the set of colors chosen is sufficiently discriminable. It is at this point that the capability of displaying colors of known chrominance becomes essential.

Displaying Colors of Known Chrominance

The requirement. As indicated previously, the specification of colors to be used to code information can come from several sources. The result of a survey of the human factors literature might be a set of color coordinates reported to be useful under certain conditions. In most cases, if the characteristics of stimuli are quantified at all, either CIE color coordinates are given (an overview of the CIE color coordinate system is given below) or color coordinates which can be transformed into CIE coordinates are given. The CIE coordinates of the colors used to produce paper maps are also available. Additionally, most metrics of color contrast, which may be used to predict the perceived contrast between two colors, are calculated using expressions which contain CIE coordinates (e.g., CIELUV-1976 contrast equations as described in Robertson, 1977). Thus, the results of most efforts to specify a set of colors to be used to color code information will be a set of CIE coordinates or a set of coordinates transformable into CIE coordinates. The general use of CIE color coordinates and the basis for their use are discussed below.

Color spaces and the CIE color coordinate system. The need for color spaces and color coordinates arose from efforts to quantify and systematize the results of color mixing experiments. Color mixing experiments, and types of color displays,

can be separated into two general categories--subtractive and additive. A color can be obtained using subtractive primaries (usually yellow, magenta, and cyan) by mixing substances that selectively reflect radiant energy in the visible spectrum. A color can be created using additive primaries (red, green, blue) by mixing the emissions of sources of radiant energy. Colors created using paints or inks result from subtractive mixtures, while colors created using lamps or phosphors bombarded with beams of electrons (color CRT) result from additive mixtures. Since CGTDs use color CRTs as the display device, and since the color CRT is an additive color mixing device, the remainder of the discussion is restricted to additive color mixing.

Early investigators (Judd, 1979, p. 453-462) found that most colors can be mixed by combining the output of red, green, and blue colored lamps. This result led quite naturally to the specification of a color, C, as the sum of its constituent primaries, or:

$$C = rR + gG + bB,$$

where:

C = Color being matched or produced

R = Red primary

G = Green primary

B = Blue primary, and

r, g, b are the respective intensities of the primaries.

The amounts of primary mixed together (r, g, b) can be viewed as color coordinates. The amount of each primary required to make a color match can be thought of as the distance along the R, G, and B axes of a color space. The matched color is located in the color space at the end of a resultant vector. This method of specifying colors, called color measurement by tristimulus values (Judd, 1979) solves the problem of specifying colors except for two limitations. First, as the spectral composition of the primaries varies, so do the color coordinates. In the case of a color CRT, as the composition of phosphors is changed, the spectral content of the emitted light changes, and the proportions required to mix given

colors change. As pointed out by Pearson (1975, sect. 6.2), transformations between sets of primaries are fairly simple as long as each new primary can be matched by a linear combination (or mixture) of the old primaries. In equation form,

$$
\begin{aligned}
R' &= r_1R + g_1G + b_1B \\
G' &= r_2R + g_2G + b_2B \\
B' &= r_3R + g_3G + b_3B
\end{aligned}
$$

or in matrix form,

$$
R' = \begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} \qquad G' = \begin{bmatrix} r_2 \\ g_2 \\ b_2 \end{bmatrix} \qquad B' = \begin{bmatrix} r_3 \\ g_3 \\ b_3 \end{bmatrix}
$$

$$
P = [R' \vdots G' \vdots B'] .
$$

The matrix P is the transpose of the matrix of coefficients of the linear equations which specify how the old primaries (R, G, B) can be mixed to match the new primaries (R', G', B'). New color coordinates (r', g', b') can be calculated from the old color coordinates (r, g, b) by using the formula,

$$
\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = P^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix} .
$$

To make the transition between sets of different primaries, one needs only to know the mixture of old primaries that matches each of the new primaries.

The second limitation of specifying colors by the mixture of the emission of lamps or the emissions of phosphors is that there will be a set of highly saturated colors (colors which appear to be very pure, or vivid) that cannot be matched by any mixture of realizable primaries (Judd, 1979, p. 456). The only way to provide a match between some of the saturated colors and mixtures of the primaries is to add one of the primaries to the color being matched and then to readjust the

mixture of the primaries. To cite an example (presented by Judd, 1979), a saturated cyan (blue-green) may not be matchable by any mixture of a blue and a green primary. The mixture of primaries may appear less saturated, or more grayish than the saturated cyan to be matched. If red primary is added to the cyan stimulus, a match using some proportion of the red, green, and blue primaries may be achieved. However, the match would be recorded as,

$$C + r'R = rR + gG + B, \qquad \text{and when simplified,}$$
$$C = -r''R + gG + B.$$

To avoid the difficulty of interpreting negative color mixture coefficients and to provide a standard set of color mixing data and a standard set of primaries, the Commission Internationale de l' Eclairage (CIE) developed a set of standard observer mixing functions (based on experimental data), and derived a set of primaries yielding nonnegative coefficients for matches of spectral colors. Figure 1 shows the amounts of each CIE primary needed to match equal energy spectral (monochromatic) colors between 400 and 700 nm. The amounts of primary

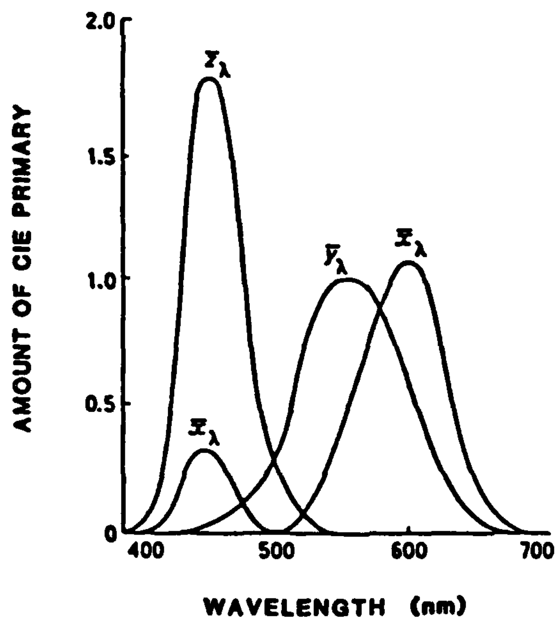


Figure 1. Relative amount of CIE primary ($\bar{x}$, $\bar{y}$, $\bar{z}$) required to match equal energy monochromatic lights of a given wavelength.

needed to make a match at a given wavelength are all positive. It is important to realize that the CIE primaries are not real, and that the energy distributions of the primaries are not physically realizable. However, mixtures of CIE primaries can be used to specify any set of real primaries. Thus, by knowing CIE color coordinates of any set of colors, and by knowing the CIE color coordinates of a set of R, G, B primaries, a set of equations can be solved to find the amounts of R, G, and B primaries to be mixed to achieve the specified color. In equation form,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} .$$

where P^{-1} is the inverse of the RGB to CIE XYZ transition matrix,[1] X, Y, Z are the CIE tristimulus values of the desired color, and r, g, b are the amounts of red, green, and blue primary required to produce a matching color. Colors are often specified by two CIE color coordinates (as opposed to tristimulus values), and the desired luminance, L. Color coordinates are simply normalized tristimulus values, given by:

$$x = X / X + Y + Z$$
$$y = Y / X + Y + Z, \text{ and,}$$
$$z = Z / X + Y + Z = 1-x-y.$$

The luminance of a color has been set equal to its Y tristimulus value. By knowing x, y, and that L = Y, we may calculate the X, Y, Z tristimulus values,

$$L = Y = y/(X + Y + Z),$$
$$Y/y = X + Y + Z,$$
$$X = x \cdot (y/Y)$$
$$Z = z \cdot (y/Y) = (1 - x - y)\, y/Y.$$

---

1 The transition matrix is the transpose of a 3 x 3 matrix of coefficients. Each coefficient corresponds to the amount of X, Y, or Z primary needed to match the R, G, or B primary.

To find the tristimulus values of any given color for which chromaticity coordinates are not known, one uses the formulas given below:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \bar{x}_{300} & \cdots & \bar{x}_{800} \\ \bar{y}_{300} & \cdots & \bar{y}_{800} \\ \bar{z}_{300} & \cdots & \bar{z}_{800} \end{bmatrix} \quad \begin{bmatrix} Le_{300} \\ \cdot \\ Le_{800} \end{bmatrix}$$

or,

$$= \begin{bmatrix} \bar{x}_w \\ \bar{y}_w \\ \bar{z}_w \end{bmatrix} \quad Le_w$$

Where,

$\bar{x}_w$, $\bar{y}_w$, and $\bar{z}_w$ are vectors of tabled values taken from the color mixing functions shown in Figure 1, Le_w is a vector of radiances at given wavelengths emitted (or reflected) by the color of interest, and w is the wavelength increment. As indicated previously, luminance (L) in the CIE system is defined as:

$$L = Y = \bar{y}_w \ Le_w.$$

We are now in a position to summarize some of the reasons why the CIE color coordinate system has gained widespread usage. The properties of the CIE color coordinate system and color mixing that are of use to display designers are:

- All realizable primaries can be specified as mixtures of the X, Y, and Z CIE primaries.

- If the CIE color coordinates and luminance of any color to be mixed and the CIE color coordinates of the primaries to be mixed are known, then the amount of each primary to be mixed to obtain a match can be calculated.

- Every perceivable color has a unique set of color coordinates.

Color CRTs and CIE color space. One of the properties of the CIE XYZ color space is that a plot of the x, y coordinates yields a bounded area (a plane of the CIE color space) which contains the coordinates of all real colors, independent of luminance, as shown in Figure 2. The boundary of the closed solid is called the spectrum locus. Spectral, or monochromatic, colors yield points that lie on the spectrum locus. As one moves from the spectrum locus toward the center of the closed solid, an area is encountered where the associated colors elicit a color name of white. Generally, as one moves progressively from the spectrum locus toward the center of the bounded area, the associated colors appear progressively less saturated. The color names associated with different regions close to the spectrum locus are given in Figure 2.

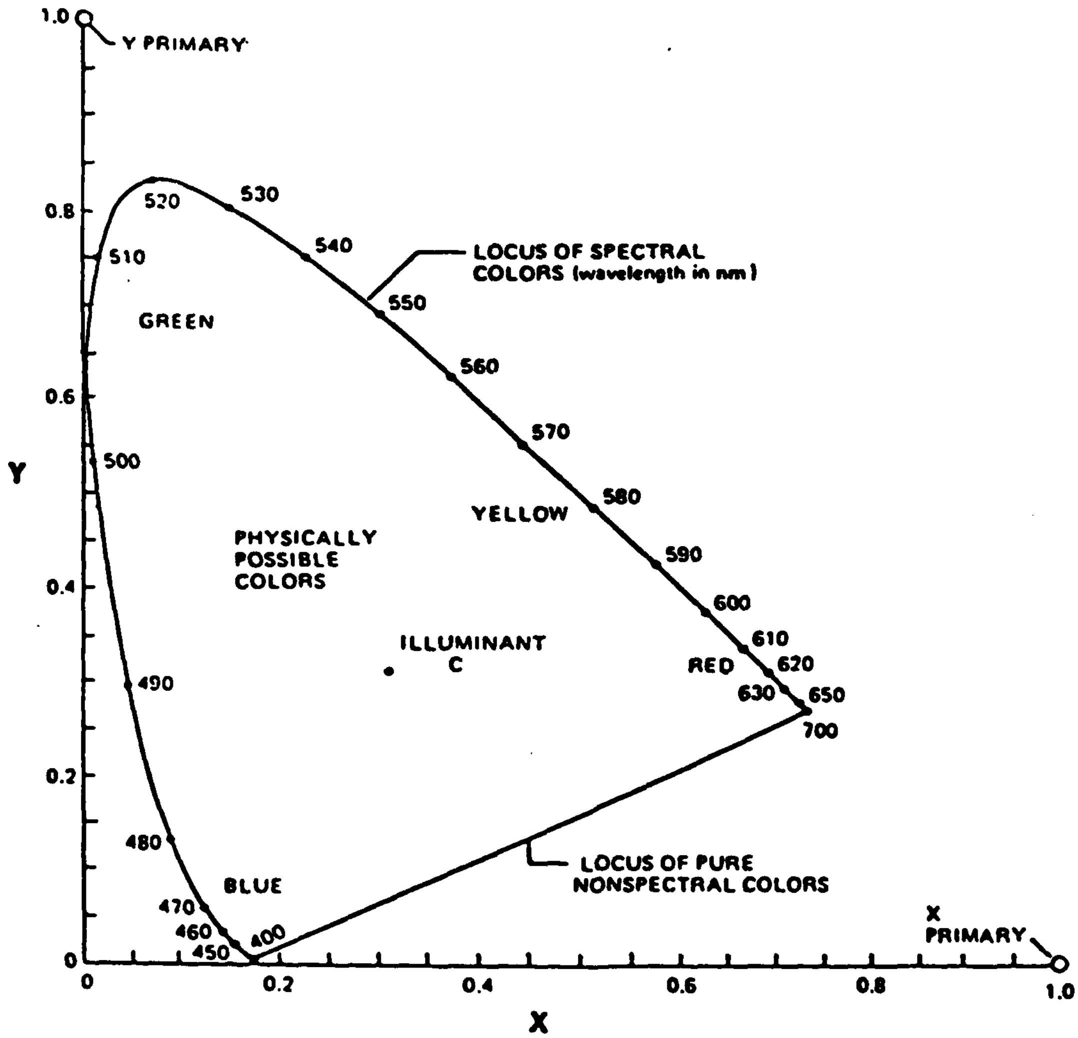


Figure 2. The x, y plane of CIE color space and a plot of the CIE color solid.

The colors that can be mixed on a color CRT are represented by a triangle, the vertices of which are the x, y CIE chromaticity coordinates of the red, green, and blue phosphors plotted on a CIE diagram as presented in Figure 3. Any color in the interior of this triangle can be mixed by some combination of the emissions of the red, green, and blue phosphors.

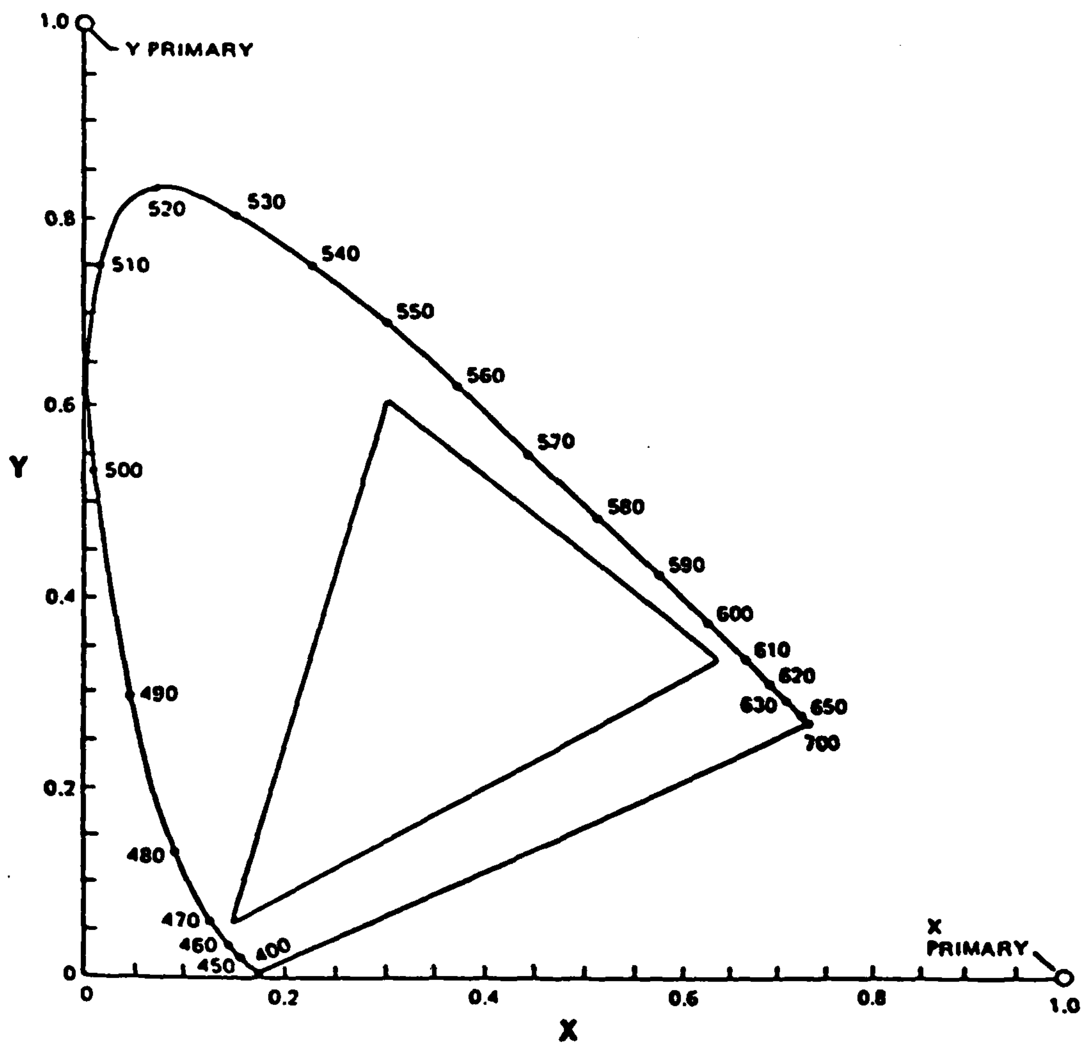


Figure 3. The x, y plane of CIE color space and the chromaticity coordinates of the emissions of typical red, green, and blue phosphors. The interior of the triangle formed by joining the points corresponding to the chromaticity coordinates of the phosphors represents the color mixing space of a color CRT.

As indicated in the previous discussion, given the x, y CIE color coordinates and the desired luminance, one can specify the amount of R, G, and B primary to be mixed to yield a match to the desired color. If the luminance output of a color CRT were proportional to the level of an R, a G, and a B input signal and if the shape of the spectral distribution of the emission of the phosphors were constant, then color mixing could be achieved by determining the appropriate proportionality constants. However, as indicated in Gutmann and Farley (1980), the input (in the case of a digital color graphics system, a triad of bits) to luminance output transfer function is nonlinear. Farley and Gutmann (1980) present a technique for obtaining a bits-to-luminance transfer function and detail a method for displaying colors of known chromaticity using a digital image processing system. Unfortunately, the method developed by Farley and Gutmann requires the use of expensive spectroradiometric equipment and requires the use of automated data collection, reduction, and analysis.

Overview of the Report

This report contains the results of an effort to develop a simple method of displaying colors of specified chrominance on a digital color graphics display. The technique does not require the use of spectroradiometric equipment. A photometer, which measures the luminance of a display, is the only measurement device that is required when using the technique discussed here.

Sections of the report detail: the equipment used to characterize the bits-to-luminance transfer function of a color graphics system of fairly common design, the results of attempts to characterize the bits-to-luminance transfer function, and a method for using the bits-to-luminance transfer function to display colors of specified chromaticity. Finally, a discussion of the assumptions and the utility of the technique is presented.

METHOD

Apparatus

The bits-to-luminance transfer function of an AED-512 (Advanced Electronics Design, Inc., Sunnyvale, California) color graphics terminal was characterized. The AED-512 had a resolution of 512 vertical by 512 horizontal picture elements (pixels). The aspect ratio of the displayed image was nearly 1-to-1. The AED-512 also had the following features: graphics firmware, stored in programmable read-only memory; down-loadable random access memory, which can store user-defined symbol sets that can be displayed using firmware capabilities; a color look-up table capable of storing up to 256 colors (specified as a triad of red bits, green bits, and blue bits); and zoom and pan capabilities. The AED-512 came equipped with a Mitsubishi Electric Corporation 13-inch diagonal color CRT (model C-3419).

A Spectraspot PR-1600 model SPRD-1° (PhotoResearch, a division of Kollmorgen Corporation, Burbank, California) was used to measure the luminance of the AED-512 color CRT display. The PR-1600 was equipped with a silicon detector, a 1° measuring field aperture, a photopic response filter, and a 55 mm f/2.8 objective lens. The absolute accuracy of the PR-1600, as calculated by the PhotoResearch calibration department, was 4.1% and the relative accuracy of the PR-1600 was estimated at between 1-2% of full-scale reading. The sensitivity of the instrument range from 1.999×10^{4} fL to 1×10^{-3} fL. The PR-1600 was equipped with a digital display capable of reading from 19.99 to 0.01. Four sensitivity scales were provided in decade increments ranging from 10^{3} to 10^{-1}.

Procedure

The PR-1600 photometer was placed 115 cm from the face of the CRT. At this distance, the edges of a 1° spot (corresponding to the photometer's measuring field aperture) covered a CRT screen area approximately 2.0 cm in diameter. The center of the CRT screen was located using a 10 x 10 grid pattern displayed on the CRT. The location of the PR-1600 was adjusted until the instrument's line of regard was perpendicular to the face of the CRT. Luminance measurements were made using a full-screen display of the color of interest. Nine sets of luminance

measurements were made over a 26-day period. The luminance of the red, green, and blue phosphors were measured separately at 16-bit increments between 32 and 256 bits. Previous measurements indicated that bit levels of less than 32 bits yielded luminances of less than .002 fL. As luminances of less than .002 fL were too small to be of practical significance, all measurements began at 32 bits.

After a change in sensitivity was made (e.g., changing from 10^0 to 10^{-1} scale), the PR-1600 required several minutes to come to a steady state reading of zero with the shutter closed. For this reason, measurements of the luminance output of the red, green, and blue phosphors were grouped together and were made before switching sensitivity scales. (A detailed description of the procedure used is presented in Appendix A.) The PR-1600 had two integration time settings, and all measurements were made using the longer integration time. A reading was not judged as stable, and hence was not recorded, unless it stayed within $\pm.02$ units of the initial reading for a period of two minutes.

RESULTS

The nine sets of measurements were analyzed to obtain models of the bits-to-luminance transfer function. The first steps taken toward selecting a model of the bits-to-luminance transfer function involved examining the general form of the relationship between bits and luminance and examining the range of the luminance measurements. Once the general form of the bits-to-luminance relationship had been characterized, classes of models were selected and fit.

Characterizing the Luminance Measurements

Figure 4 shows the nonlinear relationship between bits and mean luminance; the relationship is monotonic and increasing. Also of interest is the result that the green phosphor, at any given bit level, emits substantially more luminance than the red phosphor, which in turn emits more luminance than the blue phosphor. These results are in agreement with those found by Farley and Gutmann (1980).

Figure 5 presents the range of luminance measures expressed as percent error and calculated by:

$$\text{Upper Percent Error} = \frac{L_{max} - \bar{L}}{\bar{L}} \times 100,$$

$$\text{Lower Percent Error} = \frac{L_{min} - \bar{L}}{\bar{L}} \times 100,$$

where,

L_{max} = Maximum luminance at a given bit level,
L_{min} = Minimum luminance at a given bit level,
$\bar{L}$ = Mean luminance at a given bit level.

Luminances of less than .04 foot lamberts (fL) have been eliminated from the data and from all analyses and figures. The results of preliminary measurements taken together with the PR-1600 relative accuracy specification lead to the conclusion that measurements of less than .04 fL are neither repeatable nor accurate enough to be of practical use.

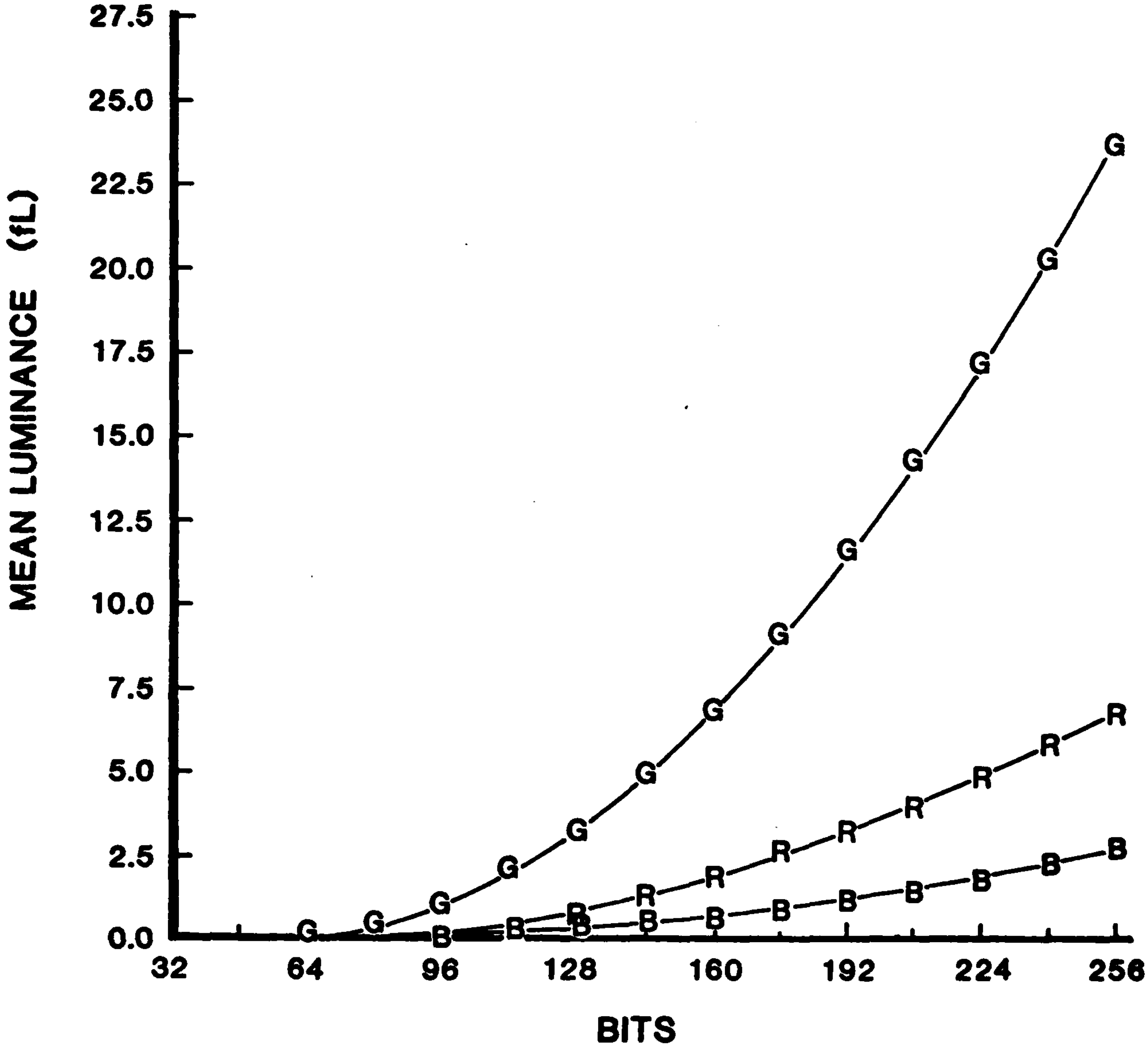


Figure 4. Mean luminance as function of bits for the red (R), green (G), and blue (B) phosphors.

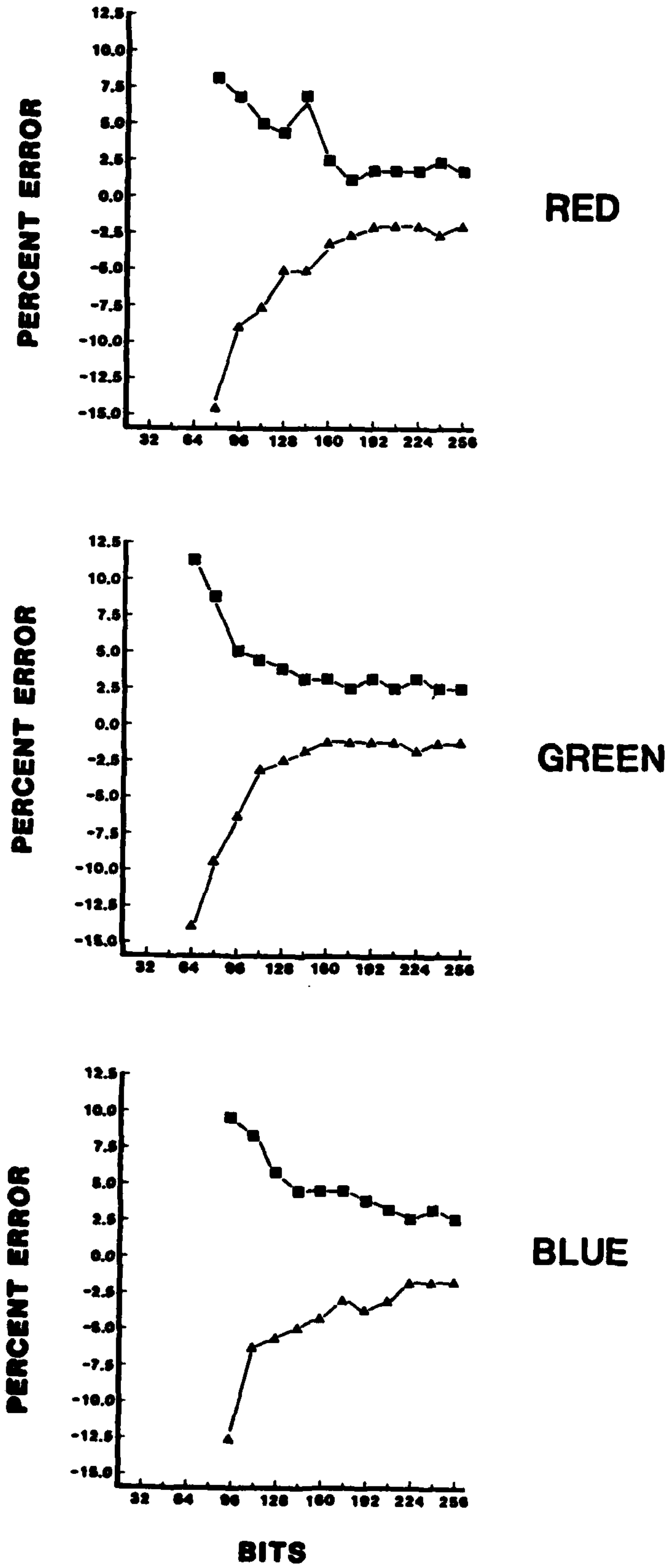


Figure 5. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors.

There are several important features that should be noted about the results presented in Figure 5. The range, expressed by percent error, appears to asymptote at approximately $\pm 2.0\%$. This result is consistent with the PR-1600 relative accuracy specification of between 1-2% of full scale reading. In all three plots, the upper percent error is slightly higher than the lower percent error. This trend is probably due to a consistent positive drift of the PR-1600. The magnitude of percent error increases as bits, and hence luminance output, decreases. This result is also consistent with the PR-1600 relative accuracy specification which is given in terms of a full scale reading. As the luminance level decreases, the contribution of measurement error to percent error values increases. Finally, the lowest bit values, and hence luminances, yield percent errors of less than $\pm 15\%$. The percent error results presented in Figure 5 represent a baseline that can be used to assess the adequacy of models of the bits-to-luminance relationship. The best predicted values a model can provide will yield percent errors identical to those presented in Figure 5. The less accurate a model is in predicting mean luminance, the higher the associated percent error will be, as calculated using the percent error formulas similar to those presented previously. The adequacy of models of the bits-to-luminance relationship can be assessed by comparing the percent error associated with a particular model with the percent error displayed in Figure 5.

Models of the Bits-to-Luminance Relationship

The General Linear Model (GLM) procedure of the Statistical Analysis System (SAS) package was used to fit second order polynomials to the bits-to-luminance relationship. The GLM procedure uses a multiple linear regression or least squares fitting approach to provide estimates of model coefficients. A second order polynomial was judged to be appropriate since the bits-to-luminance relationship is monotonically, and nonlinearly, increasing. The equations fitted are given below:

Model 1 -- $L = a(\text{Bits})^2 + i$, and

Model 2 -- $L = a(\text{Bits})^2 + b(\text{Bits}) + i$.

The resulting estimated coefficients and multiple linear regression R^2 values are presented in Table 1. As shown in Table 1, the R^2 values are quite high, and one would expect a reasonably good fit of the bits-to-luminance relationship. As can be seen from Figure 6, however, Model 1 yields high percent error values for bit values less than 112 (corresponding to luminances of .56 fL for the red phosphor, 2.07 fL for the green phosphor, and .13 fL for the blue phosphor). The finding of percent errors over 100 leads to the conclusion that this model would not be appropriate for use in color mixing.

TABLE 1
SECOND ORDER POLYNOMIAL MODELS OF LUMINANCE DATA: MEAN LUMINANCE AS A FUNCTION OF BITS

Phosphor	Model	R^2	Coefficients a	b	i
Red	$\bar{L} = a(\text{Bits})^2 + i$	.995605	1.17×10^{-4}		-9.13×10^{-1}
Green		.995848	3.94×10^{-4}		-2.59
Blue		.995138	4.99×10^{-5}		-5.39×10^{-1}
Red	$\bar{L} = a(\text{Bits})^2 + b(\text{Bits}) + i$	.999583	1.67×10^{-4}	-1.73×10^{-2}	4.15×10^{-1}
Green		.999350	5.35×10^{-4}	-4.64×10^{-2}	7.11×10^{-1}
Blue		.999248	7.60×10^{-5}	-9.29×10^{-3}	2.24×10^{-1}

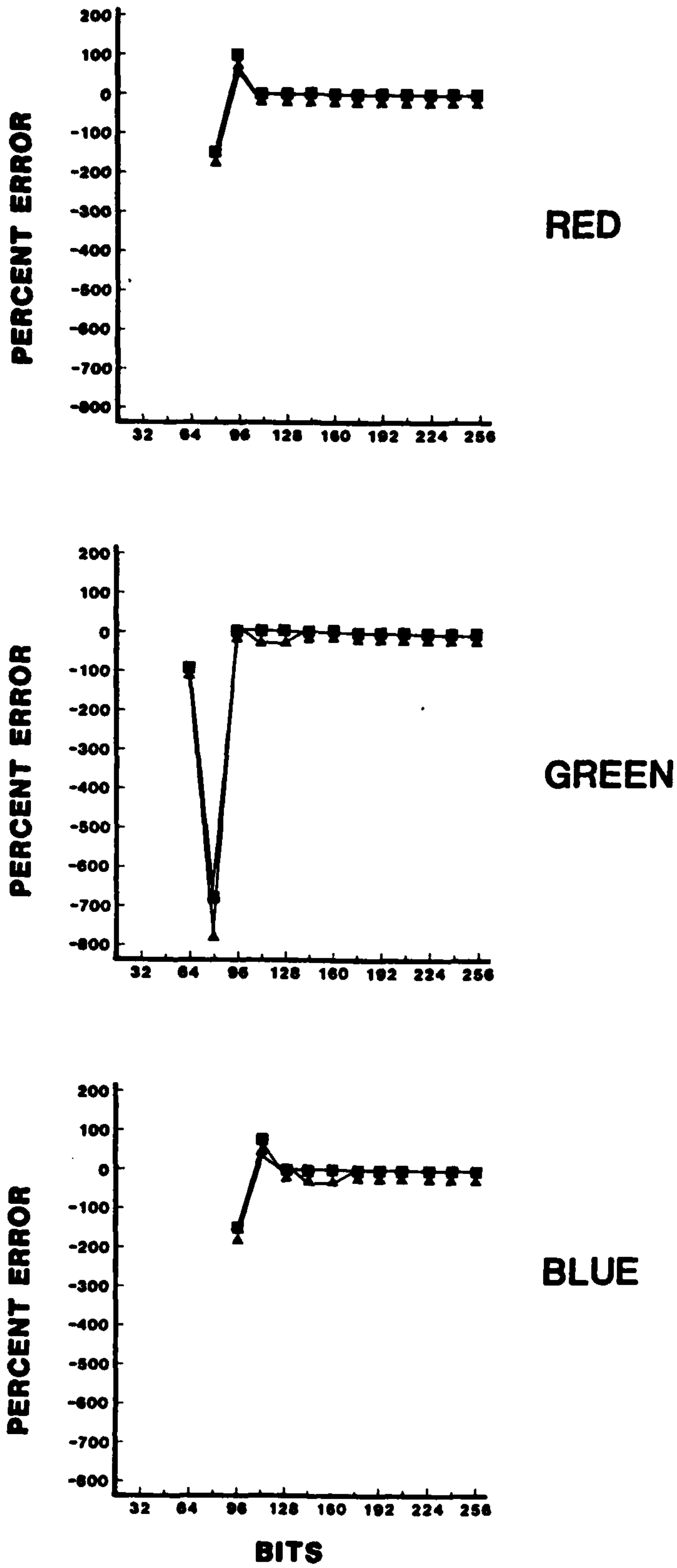


Figure 6. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$\text{Luminance} = a(\text{Bits})^2 + i.$$

The values of upper and lower percent error for Figures 6-12 are calculated using:

$$\text{Upper Percent Error} = \frac{L_{max} - \bar{L}}{\bar{L}} \times 100$$

$$\text{Lower Percent Error} = \frac{L_{min} - \bar{L}}{\bar{L}} \times 100$$

where,

L_{max} = maximum luminance at a given bit level,
L_{max} = minimum luminance at a given bit level,
$\bar{L}$ = predicted luminance, obtained from the appropriate model at a given bit level.

Upper and lower percent error values are simply the model residuals (datum minus predicted value) associated with the maximum and minimum luminances measured at a given bit value and divided by predicted value. The percent error values are calculated in the same manner as those presented in Figure 5, except for the substitution of predicted luminances for mean luminances.

As shown in Figure 7, the percent error values obtained from a full polynomial model (Model 2) are much lower for bit values smaller than 112. However, the green phosphor Model 2 yields 200-300 percent error values at 64 bits. The full second order polynomial models associated with each of the three phosphors (red, green, and blue) yield large percent errors for bit values less than 112, as indicated by a comparison of Figures 5 and 7.

The necessity to have better fits of the low end of the bits-to-luminance relationship becomes clearer when one considers mixing desaturated colors which, in an additive color mixing scheme, requires a mixture of all three primaries to produce white. White, when added to any color, will elicit the perception of desaturation or a description of the color as being less vivid or pure. Pink is an example of a desaturated red. A desaturated cyan is produced by mixing small

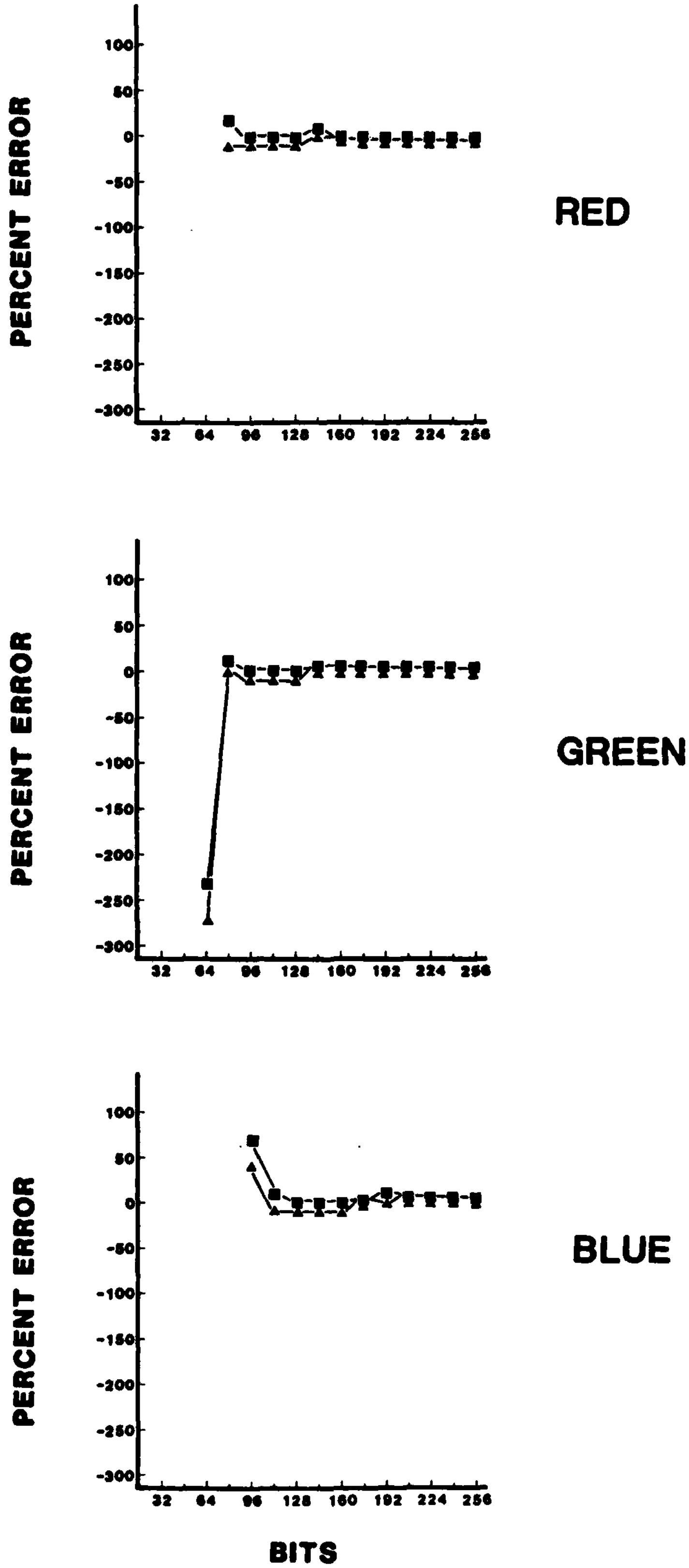


Figure 7. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$\text{Luminance} = a(\text{Bits})^2 + b(\text{Bits}) + i.$$

amounts of red, green, and blue primary to a mixture of blue and green. Without an accurate method for predicting the luminance output associated with the lower bit values, low luminance colors and desaturated colors may differ perceptibly from the desired color.

To provide a more accurate model of the bits-to-luminance relationship for low bit values, second order polynomial models were fit for bit values of less than 128. The results of the regressions are contained in Table 2. Figures 8 and 9 present the resulting percent errors. The percent error values associated with Model 1 and shown in Figure 8 are smaller than those associated with the full range fit and shown in Figure 6. The low bit value Model 1 fit yields percent errors that are approximately equal to those obtained from the full bit value fit of the full second order polynomial (Model 2) seen by comparing Figures 7 and 8. A dramatic reduction in percent error is achieved when a full second order polynomial is fit for bit values less than 128. Examination of Figures 5 and 9 reveals that the percent error values associated with the raw data and those associated with the full second order polynomial fit (for bit values less than 128) are quite comparable. The second order polynomial model provides as accurate a prediction of luminance as do calculations of mean luminance from the raw data. These results clearly indicate that a two model or spliced model will provide excellent predictions of mean luminance from bit values.

TABLE 2
SECOND ORDER POLYNOMIAL MODELS OF LUMINANCE DATA: LUMINANCE AS A FUNCTION OF BITS, FOR BIT VALUES OF LESS THAN OR EQUAL TO 128

Phosphor	Model	R^2	Coefficients a	b	i
Red	$L = a(\text{Bits})^2 + i$	.988295	8.23×10^{-5}		-4.43×10^{-1}
Green		.981142	2.71×10^{-4}		-1.21
Blue		.985781	3.03×10^{-5}		-2.35×10^{-1}
Red	$L = a(\text{Bits})^2 + b(\text{Bits}) + i$	.99629	1.92×10^{-4}	-2.30×10^{-2}	7.20×10^{-1}
Green		.998677	6.38×10^{-4}	-7.12×10^{-2}	2.05
Blue		.995435	1.03×10^{-4}	-1.63×10^{-2}	6.66×10^{-1}

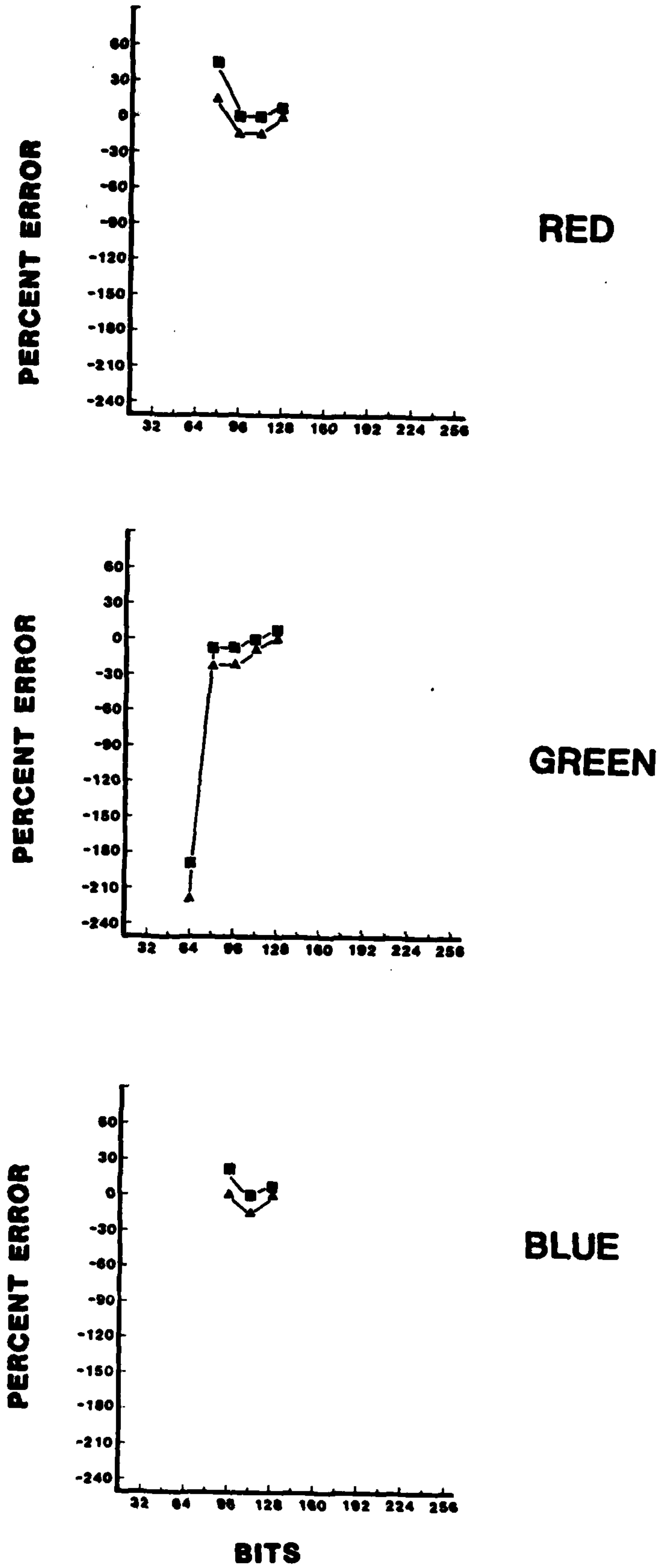


Figure 8. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$\text{Luminance} = a(\text{Bits})^2 + i,$$

for bit values of less than or equal to 128.

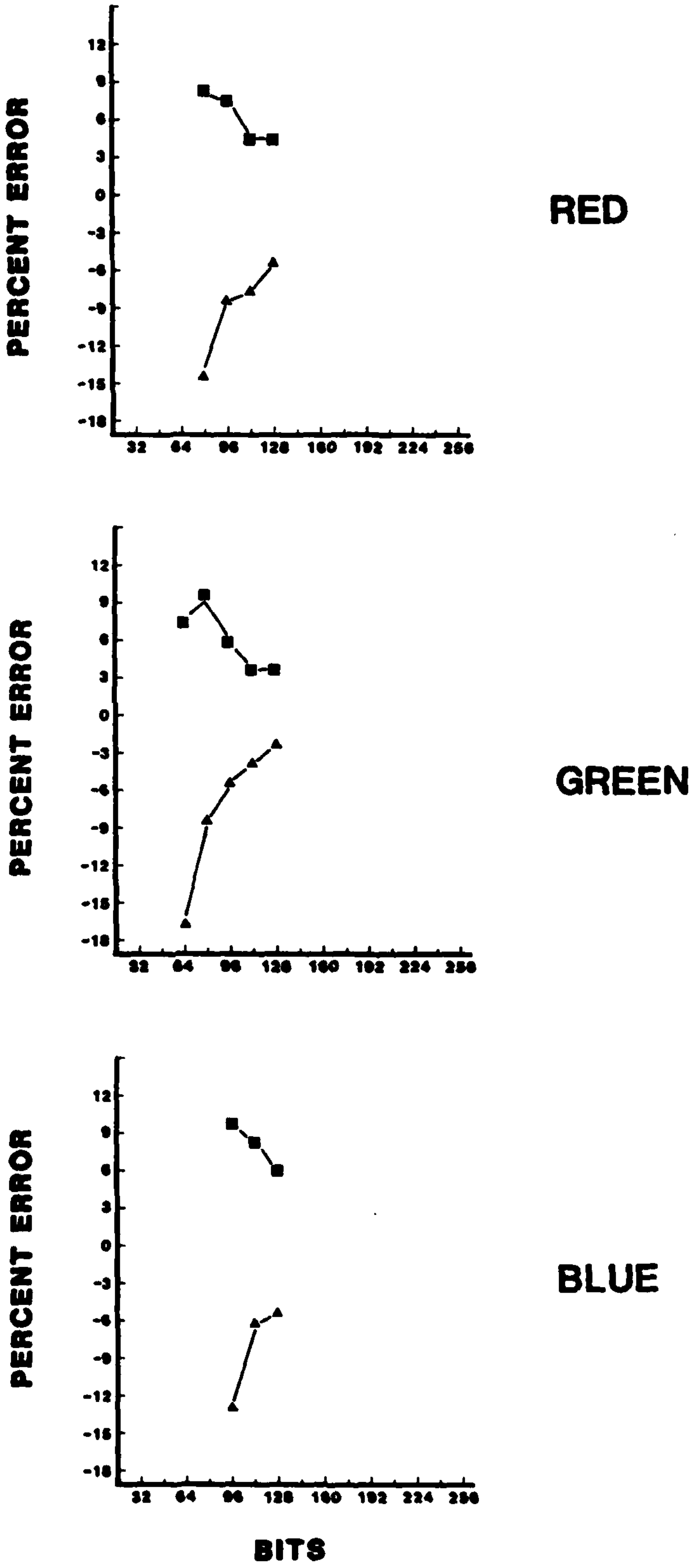


Figure 9. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$\text{Luminance} = a(\text{Bits})^2 + b(\text{Bits}) + i,$$

for bit values of less than or equal to 128.

The form of the general relationship between bits and luminance suggests that an exponent other than two might yield an appropriate fit to the data. Models of the form:

$$L = KV^n,$$

where,

L = screen luminance,

K = a constant associated with a particular display,

V = CRT screen potential,

n = constant associated with a particular display

have been used to express the relationship between the screen potential relative to the cathode and luminance (Asher and Martin, 1968). This relationship between screen potential and luminance is monotonic and increasing. The nonlinear regression (NLIN) procedure of SAS was used to estimate the parameters of a fit of the exponent for bits, as indicated below. A modified Gauss-Newton method was used to obtain estimates for coefficients of the following models:

Model 3 -- $L = a(\text{Bits})^b$,

Model 4 -- $L = a(\text{Bits})^b + i$,

Model 5 -- $L = a(\text{Bits})^b + c(\text{Bits}) + i$.

The modified Gauss-Newton method uses a truncated Taylor series and a linear regression or linear least squares technique to estimate the model coefficients in an iterative fashion. The technique described in Draper and Smith, 1968 (Section 10.2) uses the estimated model coefficients obtained from a previous iteration as starting points for the next iteration. The iterations continue until a convergence criterion is met. The SAS-NLIN default convergence criterion was used, as given by the equation below:

$$e = 10^{-8} \times (\text{current residual sum of squares}) + 1 \times 10^{-6},$$

and convergence assumed if

$$e > (\text{previous residual sum of squares}) - (\text{current sum of squares}).$$

The results of the NLIN procedure are given in Table 3 for each of the three models and for each of the three phosphors. Examination of Table 3 and Figures 10, 11, and 12 indicates that the power function models provide a better overall fit of the bits-to-luminance relationship than do the second order polynomial models, as indicated both by R^2 values and by percent error values. The best fit is provided by Model 5 which contains a fit exponential term as well as linear terms. Nevertheless, comparison of Figures 5 and 12 shows that the percent error values at the lower bit values of the red and green phosphor Model 5 fits are between two and three times that found in the original data. These results lead to the conclusion that the second order polynomial two-model or splice fit approach is more suitable from the standpoint of accuracy and simplicity of use. Undoubtedly, many other models of the bits-to-luminance relationship could be fit. However, given the low percent error achieved using the two model approach, further attempts to fit other models were unlikely to be very cost effective and were not undertaken.

TABLE 3
POWER FUNCTION MODELS OF LUMINANCE DATA:
MEAN LUMINANCE AS A FUNCTION OF BITS

Phosphor	Model	R^2	a	b	c	i
Red	$\bar{L} = a(\text{Bits})^b$	.997551	1.34×10^{-6}	2.79		
Green		.996417	8.09×10^{-6}	2.69		
Blue		.994720	1.10×10^{-7}	3.07		
Red	$\bar{L} = a(\text{Bits})^b + i$	.999509	7.95×10^{-6}	2.47		-3.48×10^{-1}
Green		.998977	4.64×10^{-5}	2.38		-1.19
Blue		.998967	2.36×10^{-6}	2.53		-2.41×10^{-1}
Red	$\bar{L} = a(\text{Bits})^b + c(\text{Bits}) + i$	.999583	1.39×10^{-4}	2.03	-1.58×10^{-2}	3.63×10^{-1}
Green		.999563	3.01×10^{-2}	1.43	-2.56×10^{-1}	4.93
Blue		.999398	2.64×10^{-1}	1.23	-9.27×10^{-2}	1.49

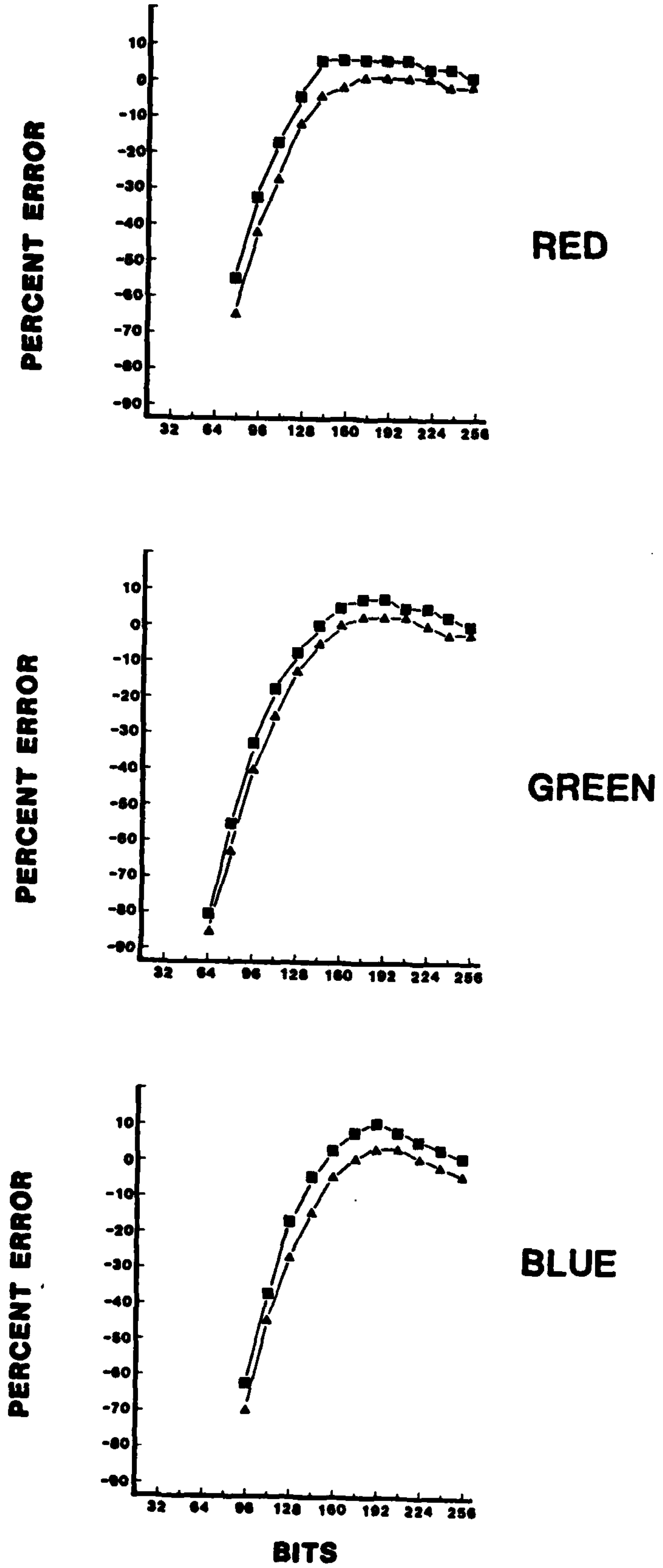


Figure 10. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$L = a(\text{Bits})^b.$$

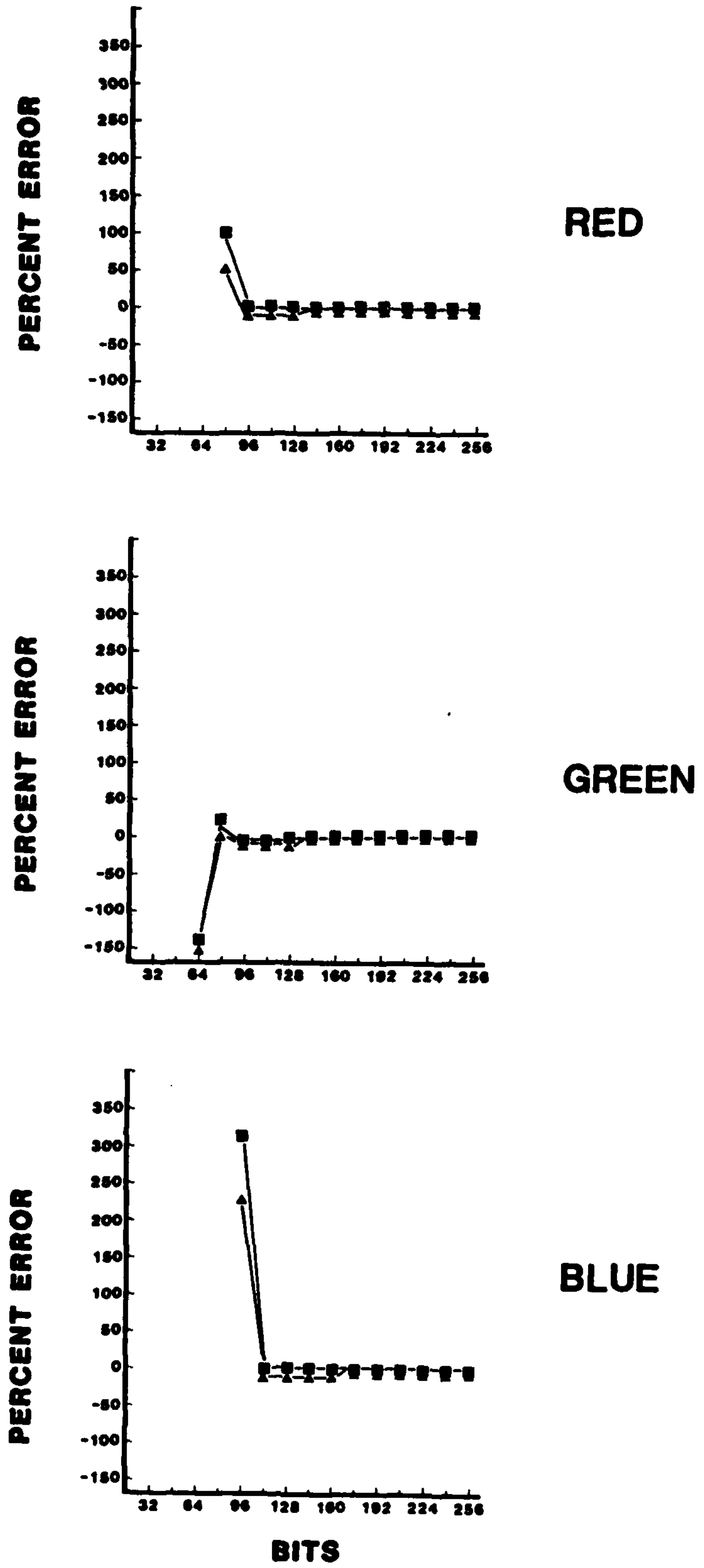


Figure 11. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$L = a(\text{Bits})^b + i.$$

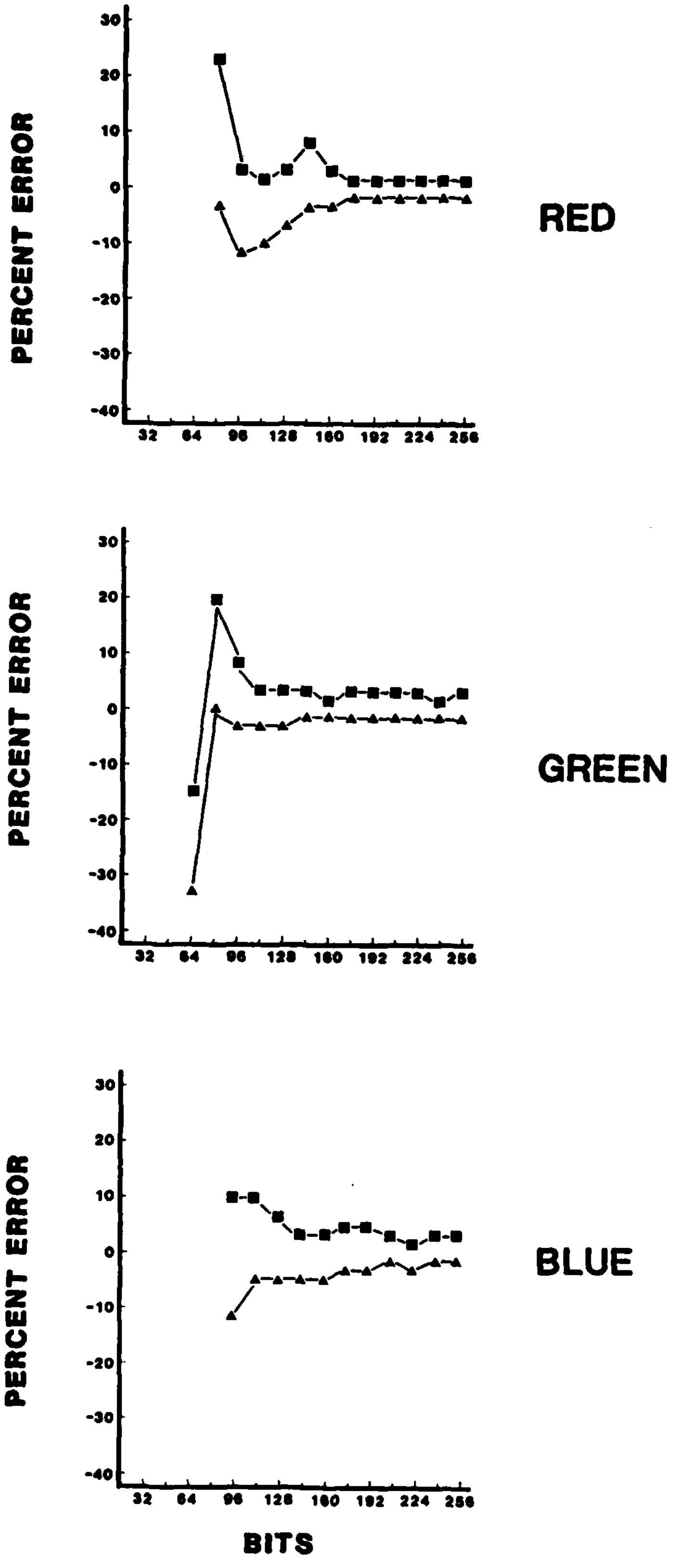


Figure 12. Upper percent error (■) and lower percent error (▲) as a function of bits for each of the red, green, and blue phosphors resulting from the model:

$$L = a(\text{Bits})^b + c(\text{Bits}) + i.$$

DISCUSSION

The finding of a nonlinear relationship between the graphics system input of bits and luminance was anticipated. However, this finding leads to a different method of specifying CRT color mixtures than is typically presented. Generally, a linear transfer function is assumed (e.g., Pearson, 1975, Chapter 7; Kuehn and Luxemberg, 1968), and the inverse of the RGB to CIEXYZ transformation matrix is used to obtain the amount of red, green, and blue primary needed to produce a match to the desired color. In the matrix notation presented earlier,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ,$$

where P^{-1} is the inverse of the RGB to CIEXYZ transformation matrix, X,Y,Z are the tristimulus values of the desired color, and r, g, b are the amounts of red, green, and blue primary to be mixed. The finding of nonlinear transfer functions necessitates modification of the color mixing equations and use of the second order polynomial models of the bits-to-luminance relationship.

Color Mixing Using a CRT

The first step in solving the problem of specifying the amounts of red, green, and blue primary to be mixed to match a given color involves expressing the emissions of the red, green, and blue phosphors in terms of CIE tristimulus values. That is:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} .$$

The symbols (X_R, Y_R, Z_R), (X_G, Y_G, Z_G), and (X_B, Y_B, Z_B) are respectively the tristimulus values of the emissions of the red, green, and blue phosphors at some specified luminance. Most often, the CIE color coordinates (x, y) of the

phosphors are known or can be obtained. However, the luminance emitted by the phosphors is variable. A method for transforming the x, y color coordinates into useful tristimulus values is needed. As noted previously, color coordinates are simply normalized tristimulus values. The color coordinates of the red phosphor can be expressed in terms of tristimulus values:

$$
\begin{aligned}
x_R &= X_R/(X_R + Y_R + Z_R), \\
y_R &= Y_R/(X_R + Y_R + Z_R), \text{ and} \\
z_R &= Z_R/(X_R + Y_R + Z_R) = 1 - x - y.
\end{aligned}
$$

Note that:

$$
\begin{aligned}
Y_R/y_R &= X_R + Y_R + Z_R, \text{ and that} \\
X_R &= x_R \cdot (Y_R/y_R), \text{ and similarly,} \\
Z_R &= z_R \cdot (Y_R/y_R).
\end{aligned}
$$

Y_R is the luminance of the red primary. In the case of a color graphics system CRT, Y_R is the luminance of the emission of the red phosphor achieved by inputting a given bit value. Substituting the relationship above, the mixing equation,

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}
$$

can be rewritten as,

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \begin{matrix} \\ = \\ = \end{matrix} \begin{bmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{bmatrix} \begin{bmatrix} (Y_R/y_R) & 0 & 0 \\ 0 & (Y_G/y_G) & 0 \\ 0 & 0 & (Y_B/y_B) \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}
$$

The tristimulus composition of a phosphor can be written in terms of its associated color coordinates and a reference luminance divided by the associated y color coordinate. All that remains is to select a meaningful reference luminance, and

the method for transforming the x, y color coordinates of each gun into tristimulus values is complete. The models of the bits-to-luminance relationship use foot-lamberts (fL) as the unit of measure. To be compatible with these models, a reference luminance of 1 fL was chosen. Although other reference luminances may be chosen (see Pearson, 1975, p. 170), a red, green, and blue reference luminance (or Y_R, Y_G, Y_B) of 1 fL yields units that are particularly useful as will be shown below.

The amount of red, green, and blue primary to be mixed to match a desired color can be found using an equation presented earlier,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} .$$

P is the transpose of the matrix of coefficients of the linear equations expressing the primaries in terms of CIE tristimulus values. Using the equations just developed and letting r = g = b = 1, we see that for the color CRT,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = P \begin{bmatrix} r \\ g \\ b \end{bmatrix} = P \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and that}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{bmatrix} \begin{bmatrix} (Y_R/y_R) & 0 & 0 \\ 0 & (Y_G/y_G) & 0 \\ 0 & 0 & (Y_B/y_B) \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} .$$

Finding P^{-1} for $Y_R = Y_G = Y_B = 1$ fL leads to the result that when the equation,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

is evaluated, the units of measure of r, g, and b are fL. The tristimulus values X, Y, Z on the right-hand side of the equation are those of the desired color, with Y being the desired luminance in fLs.

Having found the luminance of the red, green, and blue phosphors (r, g, b) needed to achieve a match to a desired color, all that remains is to determine the number of bits to input to the color graphics system to attain the desired luminances. The relationship between bits and luminance was found to be best described by an equation of the form:

$$L = a(\text{Bits})^2 + b(\text{Bits}) + i$$

By rewriting this equation as,

$$\begin{aligned} 0 &= a(\text{Bits})^2 + b(\text{Bits}) + (i - L), \\ &= a(\text{Bits})^2 + b(\text{Bits}) + c, \end{aligned}$$

and using the quadratic formula

$$\text{Bits} = \frac{-b + (b^2 - 4ac)^{\frac{1}{2}}}{2a}$$

successively for each model of the red, green, and blue phosphor bits-to-luminance relationships, the triad of bits to input to the color graphics system may be found. Due to the form of the bits-to-luminance relationship, only the root of the function found by

$$\frac{-b + (b^2 - 4ac)^{\frac{1}{2}}}{2a}$$

is needed. The root resulting from the left-hand branch of the parabola is not of interest, and in fact does not represent the measured relationship between bits and luminance.[1]

---

[1] The left-hand branch of the parabola yields increasing luminance for smaller and negative bit values. This portion of the fit curve is considered out of the domain of interest.

Two second order polynomial models of the bits-to-luminance relationship were developed. One model was fit over the domain of bits between 32 to 256. A second model was fit over the domain of bits between 32 and 128. The restricted domain model (for bit values of less than 128 bits) is used whenever the solution of the larger domain model yields a bit value of less than 128. The results of several tests showed that bit values obtained from solution of either the restricted domain or the larger domain equations were within 2 bits of each other. There was little disagreement between solutions in the region of transition between models.

The use of second order polynomial models of the bits-to-luminance relationship greatly simplifies the task of finding the triad of bit values to input to the color graphics system. (Example solutions are presented in Appendix B.) Linear interpolation between tabled values of mean measured luminance could have been used to find the triad of bit values needed to obtain desired luminances. However, the relationship between bits and luminance is nonlinear. To minimize interpolation error, more luminance measurements would have to be made over the highly nonlinear portion of the relationship. This would increase the amount of time required to calibrate a display system. Updating the linear interpolation tables would entail changing at least forty-two table entries, while updating the models would require changing eighteen coefficients (three coefficients per model for two models associated with each of the red, green, and blue phosphors). Additionally, the solution of second order polynomial equations is easier to program and verify.

Finally, the study of the variations of model coefficients from calibration to calibration may yield important insights into the nature of CRT luminance drift. A family of curves that spans the range of bits-to-luminance relationships may be identified. The identification of a family of curves could lead to a reduction in the number of measurements needed to characterize the bits-to-luminance transfer function. A small number of measurements may be sufficient to discriminate between members of the family of curves. A better understanding of CRT luminance drift should lead to better and near real time control of display chrominance.

Limitations and Assumptions of the Technique

The adequacy of the match between desired and displayed color coordinates depends on the accuracy of the models of the bits-to-luminance transfer function

and the accuracy of the measurement equipment. The accuracy of the model predictions of luminance will be compromised by a lack of stability in color CRT luminance output. Over a one-month period, drifts in luminance of between 2.5 and 24.4 percent were recorded, and over a four-month period drifts in luminance of between 7.0 and 39.9 percent were recorded. Percent drift was calculated by,

$$\text{Percent Drift} = \frac{L - \bar{L}}{\bar{L}} \times 100,$$

where L was the most recent measurement made and $\bar{L}$ was the mean of a set of previous measurements. Percent drift tends to increase as the value of bits and luminance decrease. These results indicate that color CRT luminance should be checked frequently to determine whether a different set of bits-to-luminance models is needed. Very little is known about how the drift of color CRT luminance affects the perception of chrominance. The drift of color CRT luminance will cause changes in color coordinates if the luminance output of each of the phosphors varies in a nonproportional fashion. Unfortunately, studies of color CRT luminance drift are not available. Preliminary indications based on measurements made during the course of the effort reported here are that the luminance drifts associated with each of the phosphors are not proportional. From a practical standpoint, it would be useful to know how much nonproportional luminance drift can be tolerated before the changes in chrominance affect the ability of an operator to discriminate among previously discriminable colors.

The specification of the red, green, and blue phosphor color coordinates is an important determinant of the adequacy of the match between desired and displayed color coordinates. More accurate specification of phosphor emission color coordinates leads to more accurate models of color mixing and a better match between desired and displayed colors. It is important to note that the technique presented here and presented by Pearson (1975) makes the assumption that the color coordinates of the emissions of the phosphors remain constant as luminance output is increased. The shape or form of the spectral density function (which relates radiance and wavelength) is assumed to be constant over all levels of luminance. Unfortunately, manufacturers often do not supply accurate CRT

phosphor color coordinates, and studies assessing the constancy of spectral density function shape and color coordinates are not available.

Finally, there are conditions under which colors with identical color coordinates are perceived as being different. As pointed out by Judd (1979, p. 458), the CIE color matching functions are an idealized representation of human color matching. For example, there are individual differences in sensitivity to different hues and these differences may lead to differences in the judgment of whether or not two colors match. In addition, colors that have the same (x, y) color coordinates but are mixed from primaries of differing spectral composition may not be judged as matched (Judd, 1979, p. 459). The greater the difference in the spectral composition of the primaries, the greater the probability that two colors having the same color coordinates will not be judged as matching.

Color matches can be upset by other factors. As summarized by Farrell and Booth (1975) the luminance of a target can affect the perception of both hue and saturation. If two colors have the same (x, y) color coordinates and vary only in luminance, the resulting perception should be that of two colors which match in saturation and hue, but differ in lightness or brightness. However, most studies find that the higher luminance colors are perceived as being more saturated than lower luminance colors having the same x, y color coordinates. The perception of a target color can also be altered by varying the chrominance of a surrounding field as demonstrated, for example, by Yund and Armington (1975). In this situation, colors with identical x, y color coordinates may appear quite different if the spectral content of surrounding fields differ. Chromatic adaptation can also upset color matches (Farrell and Booth, 1975).

As indicated, there are several limitations associated with the technique presented in this report. Both hardware and perceptual factors can result in a color which does not appear to match a desired or specified color. Nevertheless, the technique presented here provides a method for control of color mixing and for finer quantitative specification of colors. Given the increasing use of color displays, the capability of displaying colors of specified chrominance will be of considerable use despite the limitations mentioned above.

Summary

A simple technique for displaying colors of specified chrominance has been reviewed. Models of color CRT bits-to-luminance transfer functions and linear color mixing equations were used to find color graphic system bit values which yield colors of specified chrominance. The incorporation of models of the bits-to-luminance transfer function simplified the procedure used to solve for color graphics system bit values. The technique developed need not be limited to use with color CRTs. Once the input-to-luminance output relationship of any display has been measured and characterized, and once the color coordinates of the system's primaries are known, the technique for determining system inputs can be applied. The only restriction is that the CIE color coordinates of the display system color mixing primaries must not vary as a function of luminance output.

REFERENCES

Asher, R.W., and Martin, H. Cathode ray devices. In Luxemburg, H.R., and Kuehn, R.L. (eds.), **Display systems engineering.** New York: McGraw Hill, 1968.

Carter, R.C., Jr., **Visual search with color** (monograph NBDL-M005) Naval Biodynamics Laboratory, New Orleans, October 1980 (AD-A100 351).

Christ, R.E. Review and analysis of color coding research for visual displays. **Human Factors,** 1975 **17(6),** 542-570.

Christ, R.E. Four years of color research for visual displays. **Proceedings of the 21st Annual Meeting of the Human Factors Society,** October 1977.

Draper, N.R., and Smith, H. **Applied Regression Analysis.** New York: John Wiley & Sons, Inc., 1981.

Farley, Willard W., and Gutmann, James C. **Digital image processing systems and an approach to the display of colors of specified chrominance** (Technical Report HFL-80-2/ONR-80-2), Arlington, VA: Office of Naval Research, August 1980.

Farrell, R.J., and Booth, J.M. **Design handbook for imagery interpretation equipment.** Seattle, WA: Boeing Aerospace Company, 1975 (AD-A025453).

Judd, D.B. (MacAdam, D.L., ed.), **Contributions to color science.** (NBS Special Publication 545), Washington, D.C: National Bureau of Standards, September 1979.

Krebs, M.J., and Wolf, J.D. Design principles for the use of color in displays. **Proceedings of the Society for Information Display,** 1979, **20(1),** 10-15.

Kuehn, R.L., and Luxemburg, H.R. Visual experience and colorimetry. In Luxemburg, H.R., and Kuehn, R.D. (eds.), **Display systems engineering.** New York: McGraw Hill, 1968.

MIL-STD-1472C. **Military standard: Human engineering design criteria for military systems, equipment, and facilities.** May 1981.

Pearson, D.E. **Transmission and display of pictoral information.** London: Pentech Press, 1975.

Robertson, A.R. The CIE 1976 color difference formulae. **Color Research and Application,** 1977, **2,** 7-11.

Snyder, H.L. **Human factors analysis of flat panel display image quality** (Technical Report HFL-80-1/ONR-80-1), Alexandria, VA: Office of Naval Research, June 1980.

Yund, E.W., and Armington, J.C. Color brightness color contrast effects as a function of spatial variables. **Vision Research,** 1975, **15,** 917-929.

APPENDIX A

This appendix describes the measurement procedure used in conjunction with a PR-1600 photometer. The procedure detailed here will be of practical use to other investigators who use the PR-1600 equipped with a silicon detector, a high gain amplifier, and a digital display. The procedure was developed to minimize the effects of instrument drift which is a significant factor whenever sensitivity scales are switched. The more sensitive the scale, the longer it takes the PR-1600 to come to a steady state zero reading. The PR-1600 must be placed so that the same segment of the CRT is measured every time. It has been noted (Snyder, 1980) that the luminance of a color CRT may decrease by 50% from the center to the edge of the screen. If, from measurement to measurement, different areas of the CRT are used, additional variability will be introduced.

The procedures we have developed to counter these problems are detailed below.

1. The PR-1600 has a fairly long warm-up time. The instrument must be warmed up for at least one-half hour, and preferably for one hour.

2. The PR-1600 is located with respect to the color CRT by using a 10 x 10 grid drawn by the graphics system. The distance from the CRT to the PR-1600 is adjusted until the spot indicating the measurement area (as seen through the PR-1600 viewer) fills one square of the 10 x 10 grid. The height and lateral position of the PR-1600 must be adjusted so that the instrument's line of regard is perpendicular to the face of the CRT and parallel to the floor, and so that the line of regard intersects the grid lines at the center of the screen.

3. After the instrument has been warmed up, the zero of the instrument must be checked. The instrument controls should be set as follows:

   a. filter selector control--zero
   b. fast/slow switch (integration time)--fast
   c. sensitivity switch--zero amplifier (Z.A.)

   The zero amplifier control knob should be adjusted until a steady state reading of 0.00 is obtained for one minute.

4. Once the instrument zero has been checked, the zero for the PR-1600 high gain amplifier (which operates when the sensitivity switch is in the 10^{-1} position) must be checked. The zero for the high gain amplifier is adjusted using the DK CRNT ZERO control. The PR-1600 is equipped with a silicon detector rather than a photomultiplier tube, and there is no dark current to be zeroed. The label for the control is a misnomer. To zero the high gain amplifier, the controls should be set as follows:

   a. filter selector control--zero
   b. fast/slow switch--fast
   c. sensitivity switch--zero dark current (Z.D.)

   The zero dark current control should be adjusted until a reading of 0.00 $\pm$.02 is obtained for one minute. If measurements are to be made of luminances less than 1.99 x 10^0, then proceed to Step 7.

5. Recheck the instrument zero using the procedure described in Step 3, and proceed with Step 6 once it is completed.

6. After the instrument zeros have been checked, one can proceed with making luminance measurements by setting the controls as follows:

   a. filter selector--photometer (PHOT)
   b. viewing shutter--closed
   c. fast/slow switch--slow
   d. sensitivity switch-10^3, 10^2, 10^1, or 10^0

   Meter readings should be allowed to stabilize. If the digital readout is less than 1.99, turn the sensitivity switch to obtain a setting for a lower power of 10. Once the appropriate sensitivity range has been selected, measurements can begin. Only stable readings should be recorded. A stable reading is one that remains within $\pm$.02 of the initial reading for a period of two minutes.

7. If a sensitivity switch setting of 10^{-1} is required (a reading of less than 1.99 x 10^0 is needed) the high gain amplifier must be rezeroed. To zero the high gain amplifier, the instrument controls should be set as follows:

a. filter selector switch--zero
b. fast/slow switch--fast
c. sensitivity switch--zero dark current (Z.D.)

Adjust the zero dark current control until a digital readout of 0.00 is obtained. The high gain amplifier circuitry has a very long time constant, and it is necessary to wait ten minutes before continuing to adjust the high gain amplifier to zero. Adjustments to obtain a reading of zero should be made at the following intervals: two minutes, three minutes, and five minutes. Typically, a total of 20 minutes is the minimum time required to zero the high gain amplifier. The instrument is considered zeroed if a reading of 0.00 $\pm$.02 is obtained after a two-minute period. If the reading exceeds the tolerance of $\pm$.02, readjust the zero using the zero dark current control and restart the timing interval.

8. After the high gain amplifier has been zeroed, measurements can be made. The controls should be set as follows:

a. filter selection--photometer
b. viewing shutter--closed
c. fast/slow switch--slow
d. sensitivity switch--10^{-1}

Only stable readings should be recorded. A stable reading is one that remains within $\pm$.02 of the initial reading for a period of two minutes.

APPENDIX B

This appendix presents a step-by-step solution for the display system input bits needed to produce colors of specified chrominance. The solution makes use of the previously presented color mixture linear algebra and models of the bits-to-luminance transfer function.

The first step is to obtain the CIE (x, y) color coordinates of the display system primaries. The color coordinates of the red, green, and blue phosphors of the AED-512 graphics display were obtained from the Advanced Electronics Designs, Inc., and are presented in Table 1B.

TABLE 1B
AED-512 PHOSPHOR COLOR COORDINATES

Phosphor	x	y	z
Red	.638	.343	.019
Green	.292	.594	.114
Blue	.150	.060	.790

A matrix of color coodinates is formed as,

$$P_{cc} = \begin{bmatrix} .638 & .343 & .019 \\ .292 & .594 & .114 \\ .150 & .060 & .790 \end{bmatrix}^T = \begin{bmatrix} .638 & .292 & .150 \\ .343 & .594 & .060 \\ .019 & .114 & .790 \end{bmatrix} .$$

The matrix of color coordinates is transformed into a matrix of tristimulus values by post-multiplying by a diagonal matrix composed of,

$$C_y = \begin{bmatrix} Y_R/y_R & 0 & 0 \\ 0 & Y_G/y_G & 0 \\ 0 & 0 & Y_B/y_B \end{bmatrix} = \begin{bmatrix} 1/.343 & 0 & 0 \\ 0 & 1/.594 & 0 \\ 0 & 0 & 1/.060 \end{bmatrix} .$$

The transition matrix from RGB to CIE XYZ is written as,

$P = P_{cc} \cdot C_y$, which for the values given above is,

$$= \begin{bmatrix} 1.860 & .492 & 2.500 \\ 1 & 1 & 1 \\ .055 & .192 & 13.167 \end{bmatrix} \text{, and,}$$

$$P^{-1} = \begin{bmatrix} .720 & -.333 & -.111 \\ -.727 & 1.351 & .035 \\ .007 & -.018 & .075 \end{bmatrix} .$$

We now use P^{-1} in the equation,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} .$$

X, Y, and Z are the tristimulus values of the desired color. We find the tristimulus values from the (x, y) coordinates and the desired luminance using the relationship,

$X = x \cdot Y/y$,
$Y = L$ = desired luminance, and
$Z = (1 - x - y) \cdot Y/y$.

Example color coordinates and tristimulus values for desired luminances of 5 fL are given in Table 2B.

TABLE 2B
EXAMPLE COLOR COORDINATES AND TRISTIMULUS VALUES

Color	x	y	L(fL)	X	Y	Z
Dark Blue	.1999	.156	5	6.407	5	20.644
Light Blue 1	.1869	.2183	5	4.281	5	13.623
Light Blue 2	.195	.2183	5	4.466	5	13.438
Orange	.5003	.329	5	7.603	5	2.594

Substituting the tristimulus values of each color into the equation,

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix},$$

and performing the matrix multiplication yields the number of fLs required from the red, green, and blue phosphors. The luminances required to mix the colors presented in Table 2B are contained in Table 3B.

TABLE 3B
LUMINANCES REQUIRED TO MIX DESIRED COLORS

Color	Red Phosphor Luminance (fL)	Green Phosphor Luminance (fL)	Blue Phosphor Luminance (fL)
Dark Blue	.649	2.826	1.524
Light Blue 1	-.099	4.124	.975
Light Blue 2	.055	3.982	.962
Orange	3.521	1.316	.163

Notice that the color labeled Light Blue 1 requires a negative amount of luminance from the red phosphor. This means that the color cannot be mixed, given the color coordinates of the phosphors. The color coordinates of this light blue color would lie outside of the triangle formed by a plot of the color coordinates of the red, green, and blue phosphors in CIE (x, y) color space. It should also be observed that low luminances are often required, hence the need for accurate measurements and models of the lower portion of the bits-to-luminance relationship. Additionally, it should be noted that high luminance colors may require more luminance to be emitted by a phosphor than can be provided. In this case, the desired color cannot be mixed.

Having found the luminances needed to mix the desired colors (Table 3B), we use the quadratic formula and the model coefficients contained in Tables 1 and 2 to solve for the number of graphics system bits required to obtain the desired luminances. The quadratic formula, for the appropriate domain, is given by,

$$\text{Bits} = \frac{-b + (b^2 - 4ac)^{\frac{1}{2}}}{2a}$$

where,

$c = i$ - desired luminance, and,

a, b, and i are the appropriate coefficients of the full second order polynomial model, given by,

$$\text{Luminance} = a(\text{Bits})^2 + b(\text{Bits}) + i$$

The full domain models for each phosphor are used first. The full domain model coefficients are contained in Table 1. If a resulting bit value is less than 128, the restricted domain models are used to solve for bits. The restricted domain model coefficients are contained in Table 2. Results using the full and restricted domain models are presented in Table 4B.

TABLE 4B
BIT VALUES YIELDING DESIRED COLORS

Color	Red Bits	Green Bits	Blue Bits
Dark Blue	117*	122*	205
Light Blue 2	71*	133	177
Orange	198	100*	116*

*Restricted domain model result.

Input to the graphics system of the bit values contained in Table 4B would result, within the limits of measurement and model accuracy, in colors having the desired (x, y) color coordinates and a luminance of 5 fL.

END

FILMED